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Hot Spots and Turbulent Initial Conditions of Quark–Gluon Plasmas in Nuclear Collisions[†]

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Abstract

As a result of multiple mini-jet production, initial conditions of the QCD plasma formed in ultrarelativistic nuclear collisions may be inhomogeneous, with large fluctuations of the local energy density (hot spots), and turbulent, with a chaotic initial transverse velocity field. Assuming rapid local thermalization, the evolution of such plasmas is computed using longitudinal boost-invariant 3+1–dimensional hydrodynamics. We compare the evolution in case that the speed of sound in the plasma is constant to one resulting from an equation of state involving a strong first order transition, with a minimum of the velocity of sound as a function of energy density. We find that azimuthally asymmetric fluctuations and correlations of the transverse energy flow, $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$, can develop in both cases due to the initial inhomogeneities. Hot spots also enhance significantly high–$k_\perp$ direct photon yields. In the case with a phase transition, the hadronization surface evolves into an unusual foam-like structure. Also in that case, we find that hadronization is considerably delayed relative to the ideal gas case, just as previous studies have found for homogeneous initial conditions. The time-delay signature of a rapid cross-over transition region in the QCD equation of state (as observable via meson interferometry) is thus found to be remarkably robust to uncertainties in the initial conditions in heavy-ion reactions.

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1 Introduction

At energies $\sqrt{s} > 100$ AGeV, copious mini-jet production in central nuclear collisions is expected [1, 2, 3] to be the main source of a very dense plasma of quarks and gluons with an initial (proper) energy density an order of magnitude above the deconfinement and chiral symmetry restoration scale [4], $\epsilon_c \sim 1$ GeV/fm^3 ($T_c \simeq 160$ MeV). A large number of observable consequences of the formation of this new phase of matter have been proposed based on a wide range of dynamical assumptions [5], and experiments are currently under construction to search for evidence of that quark–gluon plasma (QGP) at the Relativistic Heavy–Ion Collider (RHIC) at Brookhaven. The observables include dilepton and hard direct photon yields, high–$p_\perp$ jets and hadrons, strangeness and charmed hadron production, identical meson pair interferometry, and collective transverse expansion. Evidently, these and other proposed signatures depend sensitively on the assumed ensemble of initial conditions as well as on the transport dynamics through the hadronization point.

In this paper we explore the dependence of several observables on the fluctuations of the initial conditions induced by mini-jet production. Our central dynamical assumption is that after a rapid thermalization time (~ 0.5 fm/c) the evolution of the plasma through hadronization can be approximated by non-dissipative hydrodynamics. We make this strong assumption to explore the *maximal* effects that the sought after thermodynamic properties of hot QCD matter may have on observables in ultrarelativistic heavy-ion collisions. Only hydrodynamics gives a direct link between observables and the fundamental QCD equation of state [4]. Finite dissipative effects generally tend to reduce hydrodynamic signatures and require further knowledge of the transport coefficients in the QGP as well. Even in the ideal hydrodynamic limit, however, the observables are sensitive to the initial formation physics of the plasma. It is this aspect of the problem that we concentrate on in this paper.

We show that in contrast to the conventional picture of QGP formation, the initial mini-jet ensemble is characterized by a wide fluctuation spectrum of the local energy density (hot spots) and of the collective flow field (turbulence). We also show that hydrodynamic evolution of that inhomogeneous, turbulent ensemble can lead to novel observable consequences including azimuthally asymmetric transverse energy fluctuations and enhanced radiance of hard probes. An especially significant result is the demonstration that the time-delay signature of the QCD phase transition, as found in previous studies [6, 7, 8] starting from homogeneous initial conditions, survives this generalized class of more unfavourable initial conditions. Meson interferometry therefore remains one of the most robust and generic probes of QGP formation given present uncertainties of the initial conditions in heavy-ion collisions.

Before elaborating on the nature of the inhomogeneous, turbulent initial conditions induced by the mini-jet production mechanism, we review first the (homogeneous) "hot-glue scenario" [2, 3] assumed in many previous calculations of the observable consequences of QGP formation in $A+A$ collisions. Mini jets are simply unresolved partons with moderate $p_\perp > 1$ GeV/c predicted from perturbative QCD. They are produced abundantly at collider energies because the inclusive cross section for gluon jets with moderate $p_\perp > p_0 = 1$ (2) GeV/c rises to a value $\sigma_{jet}(p_0) \simeq 40$ (10) mb at $\sqrt{s} = 200$ GeV, comparable to the total inelastic cross section. Evidence for mini-jet production in pp and $p\bar{p}$ reactions at collider energies has been

inferred from the systematic rise with increasing $\sqrt{s}$ of the yield of moderate–$p_\perp$ hadrons, of the central rapidity density, of the enhanced tails of multiplicity fluctuations, as well as the flavour and multiplicity dependence of the mean transverse energy of hadrons (see Refs. [9]).

In the eikonal approximation to nuclear dynamics, the total number of mini-jet gluons produced in central $A + A$ collisions is expected to be $A^{4/3} \sim 10^3$ times larger than in pp collisions since each incident projectile nucleon interacts with $\sim A^{1/3}$ target nucleons along the beam axis. This simple geometric effect leads to a high rapidity density of mini-jet gluons with $\mathrm{d}N_g/\mathrm{d}y \sim 300 - 600$ in $Au + Au$ as shown in Fig. 1a. The curves are calculated via the HIJING model [9] with shadowing and jet quenching options turned off. Comparison of the curves for $p_0 = 1$ and 2 GeV/c gives an indication of current theoretical uncertainties associated with the extrapolation from pp to AA reactions. The observed $\sqrt{s}$ systematics of the $p\bar{p}$ data are best accounted for with $p_0 = 2$ GeV/c in the HIJING model [9]. That model combines an eikonal multiple collision formalism with the PYTHIA algorithm [10] to generate exclusive hard pQCD processes with $p_\perp > p_0$ and a variant of the LUND/DPM string phenomenology [11] to model hadronization and account for the non-perturbative, low–$p_\perp$ beam-jet fragmentation. Other parton cascade Monte Carlo models, such as developed in Ref. [12], using different structure functions and hard pQCD cutoff schemes, can account for the $p\bar{p}$ data using a somewhat lower $p_0 \simeq 1.5$ GeV/c. Theoretically [1], the scale separating the perturbative QCD domain from the non-perturbative (beam-jet) domain may be as low as $p_0 = 1$ GeV/c, although no hadronization phenomenology has yet been developed with such a low scale that could account for the available data. Another source of moderate–$p_\perp$ gluons in very heavy-ion reactions has recently been proposed based on a semi-classical treatment of the non-Abelian Weizsäcker–Williams gluon fields [13]. The above uncertainties in the initial conditions on the parton level are seen in Fig. 1b to correspond to approximately a factor of two uncertainty of the transverse energy produced per unit rapidity in central $Au + Au$ collisions at RHIC energies.

Figure 1c shows that the difference between the cases $p_0 = 1$ and 2 GeV/c in the HIJING model is due to the production of approximately twice as many gluons in the moderate $p_\perp < 4$ GeV/c region for $p_0 = 1$ GeV/c. (The $p_\perp$–spectra extend to $p_\perp = 0$ because of initial and final state radiation associated with mini-jet production.) This difference is significantly smaller than the lowest order pQCD estimate would give because of the unitarized eikonal formalism used in HIJING to calculate multiple collisions and multiple jet production. For $p_0 = 1$ GeV/c the mini-jet cross section is comparable to the inelastic cross section. Due to Glauber multiple collision shadowing, the number of mini jets in that case must scale less rapidly than with the number of binary pp collisions.

In Figure 1d the hadronization mechanism of the mini-jet gluons via the string fragmentation mechanism is found to approximately double the final hadron transverse energy distribution relative to Fig. 1b. This is due to the pedestal or "string" effect and persists up to LHC energies in this model. The mini-jet gluons are represented as kinks in the beam-jet strings, and those kinks effectively boost the produced hadrons in the transverse direction. The difference between the mini-jet contribution (Fig. 1b) and the final hadronic transverse energy distribution is due to the string model implementation of beam-jet fragmentation in HIJING. That component necessarily involves non-perturbative, low–$p_\perp$

multi-particle production and is presently under experimental study via heavy-ion reactions at lower CERN/SPS energies ($\sqrt{s} = 20$ AGeV) [5]. While the string model provides an adequate phenomenology of beam-jet fragmentation at those energies, it is not obvious that it will continue to do so at RHIC and LHC. This represents a significant source of theoretical uncertainty in calculating RHIC initial conditions. We will assume in this study that the extrapolation of the beam-jet physics via string phenomenology as encoded in the HIJING model holds up to RHIC energies. Possible sources of fluctuations of the beam-jet component due, for example, to "colour rope" formation have been explored in the past [14, 15, 16]. However, at collider energies, the consequences of fluctuations due to the dominant mini-jet component have not been considered previously to our knowledge.

In the hot-glue scenario, the thermalization proper time is assumed to be a few times the mini-jet formation time $\tau_0 = \hbar/p_0 \sim 0.1$ fm/c (our units are $c = k_B = 1$). In fact, the initial pQCD mini-jet $p_\perp$–distribution is not far from thermal as can be seen in Fig. 1c, but it turns out that the gluon and quark multiplicities are below chemical equilibrium. Inelastic multi-gluon production processes are therefore essential to achieve rapid equilibration. Recent progress on radiative transport in QCD plasmas [17, 18, 19] suggests that equilibration is possible on scales less than 1 fm/c.

Taking longitudinal boost-invariant expansion into account and assuming a cylindrical distribution of matter, the proper energy density averaged over transverse coordinates at proper time $\tau > \tau_0$ is given by the Bjorken formula [20]:

$$\bar{\epsilon}(\tau) \simeq \frac{\mathrm{d}E_\perp}{\mathrm{d}y} \frac{1}{\tau \pi R^2} \quad . \tag{1}$$

For $\mathrm{d}E_\perp/\mathrm{d}y \simeq 1$ TeV from Fig. 1d, and $R = 7$ fm, this yields an order-of-magnitude estimate of $\bar{\epsilon}(\tau) \simeq 65\,[0.1\,\mathrm{fm/c}\,/\tau]$ GeV/fm^3. If only the gluon degrees of freedom are equilibrated, then the temperature of the "hot glue" at the thermalization time is $T(\tau_{th}) \simeq [\bar{\epsilon}(\tau_{th})/5.26]^{1/4} \simeq 555\,(\tau_0/\tau_{th})^{1/4}$ MeV. The evolution of the temperature after τ_{th} of course depends on the equation of state as well as the assumed viscosity [21].

In this scenario, observables of the plasma phase such as thermal dileptons or photons can be computed as discussed in [22]. Transverse expansion can also be taken into account [23] as well as more realistic equations of state with a rapid cross-over transition region [8]. For transverse expansion, the temperature field, $T(\tau, \mathbf{x}_\perp)$, acquires a dependence on the transverse coordinates. In the hot-glue scenario azimuthal symmetry is naturally assumed for collisions at zero impact parameter.

Another implicit assumption of the hot-glue scenario is that the *initial* fluid 4–velocity field, $u^\mu(x)$, vanishes in the transverse direction and that the initial flow field only reflects the Bjorken longitudinal expansion,

$$u^\mu(t, \mathbf{x}_\perp, z) = (t/\sqrt{t^2 - z^2}, 0, 0, z/\sqrt{t^2 - z^2}) \quad . \tag{2}$$

Transverse expansion is allowed to develop in the course of subsequent evolution, but initially the plasma is assumed to be quiescent in the above sense.

In this paper, we call into question the commonly assumed azimuthal symmetry and smooth radial profile of $b = 0$ collisions and the above quiescent form of the initial velocity

field. In the next section, we show that the mini-jet mechanism does not support those simplifying assumptions unless the beam-jet component is much larger than estimated with HIJING. In Section 3, the hydrodynamic evolution of inhomogeneous, turbulent initial conditions is calculated and the novel type of azimuthally asymmetric transverse shock collectivity is discussed. In Section 4 the robustness of the time-delay signature associated with a phase transition is demonstrated. In Section 5, the enhanced radiance of direct photons from hot spots is estimated. A brief summary is finally presented in Section 6.

2 The Inhomogeneous, Turbulent Glue Scenario

Inhomogeneities arise in nuclear collisions as a result of fluctuations of the number of soft and hard QCD interactions per unit transverse area. Fluctuations of the soft beam-jet component have been considered before [14, 15, 16], but at collider energies a new source of fluctuations that are induced by mini-jet production is expected to become dominant. Both types of fluctuations are strongly correlated as a function of transverse coordinates. In this paper, however, we consider fluctuations arising from only mini-jet production and treat the soft component as smooth background component of the plasma.

Each nucleon in a central $Au+Au$ collision suffers approximately $A^{1/3} \pm A^{1/6} \simeq 6 \pm 2$ inelastic collisions. Therefore, there are ~ 40 binary collisions per $\sigma_{in} \simeq 4$ fm^2. At RHIC energies, however, only a fraction $\sigma_{jet}/\sigma_{in} \simeq 1/4$ of those produce mini jets. The fluctuations of the mini-jet number density are substantial because $A^{1/3}$ remains relatively small even for the heaviest nuclei. In principle, two–nucleon correlations in the initial nuclei could reduce the above type of geometric fluctuations. However, the available data on high–$E_\perp$ production in nuclear collisions at the SPS indicates sizable fluctuations, even beyond the independent nucleon gas approximation [24].

In addition to geometric sources of fluctuations, the broad transverse momentum spectrum of mini-jet gluons in Fig. 1c further enhances the fluctuations of the energy and momentum deposited by mini jets per unit area. These two effects conspire to induce large fluctuations of the initial energy and momentum density at early times as we show below.

The spectrum of hot spots can be computed from the HIJING event list of parton transverse and longitudinal momenta $(\mathbf{p}_{\perp\alpha}, p_{z\alpha} = p_{\perp\alpha}\sinh y_\alpha)$, and their longitudinal and transverse production coordinates, $(z_\alpha = 0, \mathbf{x}_{\perp\alpha})$. The production points are taken from the initial transverse coordinates of the nucleon (diquark-quark string) in which the PYTHIA [10] subroutines of HIJING embed the gluons.

To simplify the problem we assume Bjorken boundary conditions and neglect additional fluctuations along the longitudinal coordinate. The longitudinal velocity is thus assumed to vanish at $z = 0$. With this simplification, the transverse coordinate dependence of *local* energy and transverse momentum density in the $z = 0$ plane can then be estimated from

$$\left(\begin{array}{c} \mathcal{E}(\tau, \mathbf{x}_\perp, z=0) \\ \mathbf{M}_\perp(\tau, \mathbf{x}_\perp, z=0) \end{array} \right) = \sum_\alpha \left(\begin{array}{c} 1 \\ \mathbf{v}_{\perp\alpha} \end{array} \right) \frac{p_{\perp\alpha}}{\tau} \, F(\tau p_{\perp\alpha}) \, \delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}_{\perp\alpha}(\tau)) \, \delta(y_\alpha) \quad . \tag{3}$$

The above formula takes into account the free streaming of gluons in the transverse direction via $\mathbf{x}_{\perp\alpha}(\tau) = \mathbf{x}_{\perp\alpha} + \mathbf{v}_{\perp\alpha}\tau$ where $\mathbf{v}_{\perp\alpha} = \mathbf{p}_{\perp\alpha}/p_\alpha$, as well as ideal Bjorken longitudinal expansion. The factor $F(\tau p_{\perp\alpha}) \equiv (1 + [\hbar/(\tau p_{\perp\alpha})]^2)^{-1}$ is an estimate of the formation probability

[18] of partons with zero rapidity. High–$p_\perp$ gluons are produced first and lower–$p_\perp$ gluons later according to the uncertainty principle. Before $\tau \sim \hbar/p_{\perp\alpha}$, those $p_{\perp\alpha}$–components of the radiation field and the evolving transient Weizsäcker–Williams field of the passing nuclei [13] interfere strongly and cannot be treated as a kinetic gas of partons or a plasma. The exact form of F is not important except for extremely small times $\tau < 0.5$ fm/c prior to thermalization.

The above expression, when averaged over transverse coordinates, reduces to the original Bjorken estimate, eq. (1) with $\langle \mathcal{E}(\tau) \rangle \simeq \bar{\epsilon}(\tau)$. In addition, the averaged transverse momentum density and hence the flow field vanishes, $\langle \mathbf{M}_\perp(\tau) \rangle = 0$, up to finite multiplicity fluctuations.

To study the transverse coordinate dependence of the initial conditions given discreet mini-jet phase space coordinates, we must specify the transverse, $\Delta r_\perp$, and longitudinal, Δy, resolution scales corresponding to an elementary fluid cell. The densities, coarse-grained on that resolution scale, are obtained from (3) by the simple substitution

$$\delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}_{\perp\alpha}(\tau))\, \delta(y_\alpha) \to \frac{\Theta(\Delta y/2 - |y_\alpha|)}{\Delta r_\perp^2\, \Delta y} \prod_{i=x,y} \Theta(\Delta r_\perp/2 - |x_{i\alpha}(\tau) - x_i|) \ . \tag{4}$$

The uncertainty principle limits how small $\Delta r_\perp > \hbar/\Delta p_\perp$ could be. However, in our dynamic problem, causality restricts how large it can be. At a given proper time τ, after the two highly contracted nuclei pass through each other, the local horizon for any gluon in the comoving ($y_\alpha = 0$) frame only has a radius $c\tau$. Thus, at the thermalization time, τ_{th}, each gluon can be influenced by only a small neighbourhood of radius $c\tau_{th} \simeq 0.5$ fm of the plasma. In order to *minimize* fluctuations, we will take the transverse resolution scale to be the maximal causally connected diameter, $\Delta r_\perp = 2c\tau_{th} \simeq 1$ fm. Also, we take the rapidity width to be $\Delta y = 1$, since gluons with larger relative rapidities tend to be produced too late (due to time dilation) to equilibrate with gluons with $y = 0$. Note that the number, $N(\tau) = [R/(c\tau)]^2$, of causally disconnected domains in a nuclear area, πR^2, is initially very large and even at the onset of hadronization, $\tau_h \sim 3$ fm/c, several disconnected domains remain.

In Figure 1d we saw that at $\sqrt{s} = 200$ AGeV, the HIJING model predicts that approximately half of the final produced transverse energy arises from beam-jet string fragmentation. As emphasized before, it is unknown whether the string model coupling between the beam-jet fragmentation and mini-jet fragmentation, that accounts so well for $p\bar{p}$ multi-particle data [9], can be applied to collisions of heavy nuclei. Effective string degrees of freedom may be quenched due to the colour conductivity of the plasma [25], but soft field fluctuations associated with beam-jet fragmentation are likely to persist and contribute a background source of low–$p_\perp$ quanta at some level. In the present study, we do *not* hadronize the partons via this default JETSET string mechanism in HIJING but only use their phase space density to compute the mini-jet contribution to the initial energy–momentum tensor $T^{\mu\nu}(x)$ as above. Since our dynamical assumption is the applicability of hydrodynamics, we model the soft beam-jet component by a homogeneous background fluid. We estimate the energy density of that component from the HIJING final hadronic $\mathrm{d}E_\perp/\mathrm{d}y$. One advantage of the hydrodynamic approach is that by treating the beam-jet component as a fluid, we need not specify further details of its uncertain microscopic nature. In our case, the main function of that background fluid is to *reduce* the energy density fluctuations induced by the mini

jets. The transverse coordinate distribution of that soft component is assumed to reflect the density of wounded nucleons, i.e., it is taken proportional to $(1-[r_\perp/R]^2)^{1/2}$.

If we take into account the relatively long formation time of soft, $\langle p_\perp\rangle \simeq 0.3$ GeV/c, partons, only about half of the $\mathrm{d}E_\perp^{soft}/\mathrm{d}y \simeq 0.5$ TeV from Fig. 1d contributes to the soft background. In that case, this soft component adds a relatively small, smooth contribution ~ 5 GeV/fm^3 to the central energy density at $\tau_{th} = 0.5$ fm/c.

In Figures 2a,b the initial energy and momentum density profile of a typical central $Au + Au$ event is shown as a function of the transverse coordinates in the $z = 0$ plane. This corresponds to a HIJING event with the default mini-jet cutoff scale $p_0 = 2$ GeV/c. The profile for $p_0 = 1$ GeV/c looks similar except the energy density scale increases by approximately a factor of two. The striking feature illustrated by the lego plot in Fig. 2a is the existence of several prominent "hot spots" with $\mathcal{E} > 20$ GeV/fm^3 separated by $\sim 4-5$ fm. In this event the hottest spot reaches an energy density of about 40 GeV/fm^3. Between the hot spots are cooler regions with energy density down to the soft background scale of ~ 5 GeV/fm^3.

The turbulent nature of the initial conditions is illustrated in Fig. 2b. An arrow plot representation of the transverse momentum field is shown. The highest momentum regions tend to coincide with the regions of highest energy density. The initial transverse flow velocities are found to be distributed broadly up to $\sim 0.5\,c$. We note that turbulence here is not inherent to QGP evolution since the Reynolds number of the QGP, $\mathcal{R}e \sim R/l \sim 10$, is not large. Thus, laminar flow is not expected to break up into complex vortex structures. In our case, the turbulence of the QGP initial conditions is *induced* by the external mini-jet production mechanism. This type of turbulence is analogous to flow induced by the blades of a mixer in a bowl of liquid.

In Figure 2c the distribution of energy densities is shown, coarse-grained over 1 fm^3 cells and averaged over 200 HIJING events. Only cells in the central region with $r_\perp < 4$ fm are considered in this histogram to reduce fluctuations from the less interesting surface region. The event-averaged energy density $\langle\mathcal{E}(\tau_{th})\rangle \simeq 12$ GeV/fm^3 includes the soft background contribution discussed above. However, the distribution is highly asymmetric with relatively high probability of large and small fluctuations. The rms relative fluctuation of the initial energy density is found to be $\Delta\mathcal{E}/\langle\mathcal{E}\rangle \simeq 0.7$ for this assumed soft background level.

In Figure 2d the distribution of the effective local gluon temperature, $T_{\mathrm{eff}}(\tau_{th},\mathbf{x}_\perp) = (\mathcal{E}(\tau_{th},\mathbf{x}_\perp)/5.26)^{1/4}$, corresponding to Fig. 2c is shown. (This estimate of the temperature neglects collective flow velocities and that the gluon number as computed in HIJING is not in chemical equilibrium.) The local temperature is seen to fluctuate around the mean ~ 350 MeV with an rms width of $\Delta T \simeq 60$ MeV. It is clear from the above that the mean values of $\mathcal{E}$ and T do not provide an adequate characterization of such an ensemble of initial conditions. The ensemble of mini-jet initial conditions has a broad fluctuation spectrum primarily because the local event horizon is so small at early times. Fluctuations of $\mathcal{E}$ in finite volumes arise even in an ideal Stefan–Boltzmann thermal ensemble with $\langle\epsilon\rangle = KT^4$. A simple estimate of the magnitude of thermal fluctuations is given by $\Delta\epsilon/\langle\epsilon\rangle \simeq 2/(KT^3V)^{1/2} \simeq 0.5$ for $T = 350$ MeV, $K \simeq 5.26$ (if only the gluon degrees of freedom are assumed to be equilibrated), $V = 4(c\tau_{th})^3$ (for $\Delta y = 1$). This is comparable to the fluctuations induced dynamically by the mini-jet formation. The spectrum of fluctuations differs from an ideal

thermal one because they are driven here by Glauber nuclear geometry and the pQCD mini-jet spectrum.

In Figure 3a the smooth, event-averaged energy density profile is compared to the fluctuating energy density profiles of three other separate HIJING events. The azimuthally symmetric, event-averaged surface corresponds to the one usually assumed in the hot-glue scenario. The three individual events in parts b–d, on the other hand, show that the dynamical fluctuations extend up to 40 GeV/fm^3 (see Fig. 2c) and cause strong deviations from the smooth, event-averaged profile. The shaded contour plots above the surface plots provide another representation of the azimuthally asymmetric inhomogeneities caused by mini-jet production at the event-by-event level.

It is important to note that the hot spots are not due to isolated hard pQCD jets, but rather to the accidental coincidence of several moderate–$p_\perp$ (see Fig. 1c) mini jets in the same 1 fm^3 volume. This is seen by comparing the gluon number density profiles in Fig. 4 to the corresponding energy density profiles in Fig. 3. The typical gluon density is seen to be about 5–10 fm^{-3} at this early time $\tau_{th} = 0.5$ fm/c. This includes, in addition to mini jets, softer gluons from initial and final state radiation as well as the soft partons from the beam-jet fragmentation component.

Again we emphasize that Figs. 2–4 correspond to the minimal fluctuations from the point of view of hydrodynamic evolution. Only at later times, when the energy density is significantly lower, can the fluctuations be significantly reduced by coarse graining on much larger resolution scales. The dilemma is that *if* indeed the local thermalization is so short as hoped for in the hot-glue scenario, then inhomogeneous and turbulent initial conditions must be considered when calculating signatures of QGP formation. On the other hand, if the thermalization time turns out to be much longer than current estimates, then hydrodynamics is of course inapplicable and the observables sensitive to the initial "hot" phase of the reaction will not provide information about the sought after *thermal* properties of the dense QGP. For our purpose of exploring optimal signatures associated with the thermal properties of dense matter, we must rely on rapid thermalization and therefore live with inhomogeneities in calculating plasma observables. Inevitably this means that many observables will depend strongly on the precise form of the ensemble of initial conditions. The mini-jet initial conditions with a string model beam-jet background represent a particular ensemble obtained in extrapolating present *pp* phenomenology to nuclear collisions via the HIJING model. We will refer to the above ensemble of initial conditions as the "turbulent-glue scenario" to contrast it with the simpler initial conditions assumed in the hot-glue scenario.

Examples of plasma probes sensitive to the highest temperature phase are high-mass dilepton pairs, direct photons, and heavy quark production. They are exponentially sensitive to the fluctuation spectrum of local temperatures because of the Boltzmann suppression factor, $\exp(-M_\perp/T)$. The hot-glue and turbulent-glue scenarios differ considerably in the predicted yields of such observables because the ensemble average of hydrodynamically evolved turbulent initial conditions and the hydrodynamic evolution of ensemble-averaged initial conditions are not equivalent.

3 Hydrodynamic Evolution of the QGP

3.1 The SHASTA Algorithm and Tests

In order to explore how such turbulent plasmas may evolve, we solve numerically the hydrodynamic equations given the above ensemble of initial conditions. Hydrodynamic evolution of homogeneous plasmas has already been studied extensively [8, 23]. In most previous studies, azimuthal (cylindrical) symmetry was assumed to simplify the problem. One exception is the study of homogeneous but azimuthally asymmetric initial conditions produced in non-central nuclear collisions [26]. Our study focuses exclusively on $b = 0$ collisions, where cylindrical symmetry of *ensemble-averaged* observables must always hold. At the event-by-event level, azimuthal symmetry is of course broken by the probabilistic nature of hadronic interactions. However, we are not interested in statistically uncorrelated fluctuations, rather only in the azimuthally asymmetric correlations that can evolve dynamically out of specific inhomogeneous mini-jet initial conditions.

We assume longitudinal boost-invariance, i.e., inhomogeneities in the rapidity dimension are neglected in this first study. Mini jets from HIJING actually lead to hot-spot fluctuations that extend only one unit in rapidity. The true 3–dimensional inhomogeneities are therefore significantly larger than what will be studied here assuming that the fluid density is *constant* along fixed proper time surfaces $\tau = \sqrt{t^2 - z^2}$. The full treatment of the problem will require much more elaborate 3–dimensional simulations in the future. At present, a 3–dimensional code is in the development phase.

A very important advantage of using a hydrodynamic approach is that hadronization can be taken into account using the non-perturbative equation of state, $p(\epsilon)$, as deduced from lattice QCD simulations. The price paid for hydrodynamics is of course the necessary assumption that the equilibration rate is large compared to space–time gradients of the fluid $T^{\mu\nu}$–field.

In order to compute the evolution of turbulent initial conditions we solve the equations of relativistic hydrodynamics,

$$\partial_\mu T^{\mu\nu} = 0 \ . \qquad (5)$$

We rely on the most recent optimistic estimates [17, 18, 19] that suggest that the local equilibration time in a QGP may be short enough to neglect dissipative effects. In that case, $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ is the energy–momentum tensor for an ideal fluid, where ϵ is the (proper) energy density and p the pressure in the local rest frame of a fluid element moving with 4–velocity $u^\mu = \gamma(1, \mathbf{v})$ in the computational frame ($\mathbf{v}$ is the 3–velocity, $\gamma = (1-\mathbf{v}^2)^{-1/2}$, and $g^{\mu\nu} = \mathrm{diag}(+,-,-,-)$ is the metric tensor). The equations (5) are closed by specifying an equation of state $p(\epsilon)$.

Since we assume boost-invariance in $z-$direction, it suffices to solve the equations of motion at $z = 0$. Furthermore, since boost-invariance implies $v^z \equiv z/t$, the four eqs. (5) can be simplified to yield the three equations (valid at $z = 0$)

$$\begin{aligned}
\partial_t\, \mathcal{E} + \partial_x \left[(\mathcal{E} + p)v^x\right] + \partial_y \left[(\mathcal{E} + p)v^y\right] &= -\,F(\mathcal{E}, p, t) \ , \\
\partial_t\, M^x + \partial_x \left(M^x v^x + p\right) + \partial_y \left(M^x v^y\right) &= -\,G(M^x, t) \ , \\
\partial_t\, M^y + \partial_x \left(M^y v^x\right) + \partial_y \left(M^y v^y + p\right) &= -\,G(M^y, t) \ ,
\end{aligned} \qquad (6)$$

where $F(\mathcal{E}, p, t) \equiv (\mathcal{E} + p)/t$, and $G(M, t) \equiv M/t$. (Our notation is $T^{00} \equiv \mathcal{E}$, $T^{0i} \equiv M^i$, $i = x, y$.) These equations are solved numerically via a two–step operator splitting method. The first operator splitting step decomposes the problem into solving the above system of equations with $F = G = 0$ (this corresponds to purely two–dimensional fluid motion), and then updating the obtained solution $\tilde{\mathcal{E}}$, $\tilde{M}^x$, $\tilde{M}^y$, $\tilde{p}, \ldots$ according to the ordinary differential equations

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = -F(\mathcal{E}, p, t) \ , \quad \frac{\mathrm{d}M^i}{\mathrm{d}t} = -G(M^i, t) \ , \quad i = x, y \ , \tag{7}$$

i.e., more explicitly one corrects

$$\mathcal{E} = \tilde{\mathcal{E}} - F(\tilde{\mathcal{E}}, \tilde{p}, t)\, \mathrm{d}t \ , \quad M^i = \tilde{M}^i - G(\tilde{M}^i, t)\, \mathrm{d}t \ , \quad i = x, y \ . \tag{8}$$

This method was originally suggested by Sod [27] and was proven to be adequate for treating the analogous, azimuthally symmetric boost-invariant problem in [8].

The solution to (6) with $F = G = 0$ itself involves another operator splitting step, i.e., one first solves the system of equations

$$\begin{aligned} \partial_t\, \mathcal{E} + \partial_x \left[(\mathcal{E} + p) v^x\right] &= 0 \ , \\ \partial_t\, M^x + \partial_x \left(M^x v^x + p\right) &= 0 \ , \\ \partial_t\, M^y + \partial_x \left(M^y v^x\right) &= 0 \ , \end{aligned} \tag{9}$$

corresponding to transport of the hydrodynamic fields in $x-$direction only, and with the solution to this set of equations one solves

$$\begin{aligned} \partial_t\, \mathcal{E} + \partial_y \left[(\mathcal{E} + p) v^y\right] &= 0 \ , \\ \partial_t\, M^x + \partial_y \left(M^x v^y\right) &= 0 \ , \\ \partial_t\, M^y + \partial_y \left(M^y v^y + p\right) &= 0 \ , \end{aligned} \tag{10}$$

corresponding to transport in $y-$direction only. Equations (9) and (10) are solved with the phoenical SHASTA algorithm [28] as presented in [29], with half-step updating of the source terms and simplified source term treatment. The transport steps (9) and (10) are alternated between successive time steps to minimize systematical errors in the propagation. Per each time step, the fields are propagated according to (9) and (10), and finally corrected with (8).

The fluid evolution is studied for two idealized forms of the QGP equation of state. One is the ideal relativistic gas case

$$p(\epsilon) = \epsilon/3 = K\, T^4/3 \ . \tag{11}$$

Here, $K \simeq 5.26$, corresponding to an equilibrated gluon gas. In the second case, a Bag model equation of state with a strong first order transition at a critical temperature T_c is assumed:

$$p(\epsilon) = \begin{cases} (\epsilon - 4B)/3 & \text{if } \epsilon > \epsilon_Q \ , \\ p_c & \text{if } \epsilon_Q \geq \epsilon \geq \epsilon_H \ , \\ \epsilon/3 & \text{if } \epsilon_H > \epsilon \ . \end{cases} \tag{12}$$

With the Bag constant B and ratio of effective plasma and hadronic degrees of freedom $r = K_Q/K_H$ fixed, the energy density of the mixed phase is bounded by

$$\begin{aligned} \epsilon_Q &= \frac{4r-1}{r-1}\, B \\ \epsilon_H &= \frac{3}{r-1}\, B \ . \end{aligned} \tag{13}$$

We take here $B = 0.39691$ GeV/fm^3 and $r = 37/3$. For this choice, $\epsilon_Q \simeq 1.7$ GeV/fm^3 and $\epsilon_H \simeq 0.1$ GeV/fm^3, the critical pressure is $p_c = \epsilon_H/3$ and the critical temperature is $T_c \simeq 169$ MeV. Hydrodynamics with a more realistic equation of state with a finite cross-over region $\Delta T/T_c \sim 0.1$ was considered in Ref. [8]. In the present exploratory study, only the two idealized forms above will be contrasted.

The evolution equations are solved on a two–dimensional cartesian 200×200 mesh with grid spacing $\Delta x = 0.2$ fm. The Courant–Friedrichs–Lewy number is taken as $\lambda \equiv \Delta t/\Delta x = 0.4$. The cartesian grid breaks the isotropy of space and might lead to instabilities. As a first check to determine whether our multi-dimensional algorithm tends to produce such numerical artifacts, we consider the expansion of a cylindrically symmetric Gaussian hot spot with radius 2 fm and peak energy density $\mathcal{E} = 30$ GeV/fm^3 at rest. The time evolution should respect the initial cylindrical symmetry of the problem. In Figure 5a we show the initial (calculational frame) energy density profile ($T^{00} \equiv \mathcal{E}$). Figure 5b shows the energy density profile after evolving the hydrodynamical equations with the standard version of the SHASTA algorithm as described in [29] and the ideal gas equation of state (11) at time $t = 14.9$ fm/c (for the sake of clarity we show the profile for positive x, y only, the other quadrants are symmetric). The observed strong fluctuations which break cylindrical symmetry are due to the following: the flux limiter in our version of the SHASTA (cf. eq. (18) of [29]) prevents the occurrence of unphysical extrema *only along* the direction of propagation, i.e., the $x-$ and $y-$direction. Off the grid axis, small perturbations are not smoothed out and can grow to produce the features seen in Fig. 5b.

One way to cure this is to use a multi-dimensional flux limiter as proposed by Zalesak [30]. Here, however, we choose the simpler method of reducing the coefficient in front of the antidiffusion fluxes (default value 1/8, cf. eq. (17) of [29]; the necessity to reduce this coefficient occurs also in other, purely one–dimensional situations, cf. the discussion in [29, 31]). Figure 5d shows the same situation after evolving with the antidiffusion coefficient reduced to 70% of its default value, i.e., to 0.7/8=0.0875. This obviously strongly reduces symmetry breaking. Although this prescription increases the numerical diffusion of the algorithm and thus produces physical entropy, all our following results are generated with this reduced antidiffusion in order to avoid spurious azimuthal asymmetries of the transverse energy, $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$, of purely numerical origin. In Figure 5c we show the energy density after evolving with the equation of state with the first order phase transition (12). Also in this case, the reduction of the antidiffusion helps to preserve the cylindrical symmetry of the problem.

Another test of the algorithm is to check how well it reproduces test cases with analytically known solutions. One such test would be the expansion of a cylinder with a sharp surface. Here, however, we focus instead on the more physically relevant problem

of the expansion of the cylindrically symmetric (smooth) Gaussian studied in the previous figure. Although that problem is effectively one–dimensional, it does not have a purely analytical solution. We may, however, compare the numerical solution generated with our multi-dimensional SHASTA algorithm to that obtained with a one–dimensional algorithm which is known to reproduce analytical solutions very well, namely the relativistic Harten–Lax–van Leer–Einfeldt (RHLLE) algorithm [29, 31, 32, 33], for our purposes modified with a Sod operator splitting step as described in [8] to account for longitudinal boost-invariance (that operator splitting is in fact analogous to the one in eq. (8)).

In Figure 6 we compare energy density profiles along the $x-$axis for (a) the ideal gas equation of state and (b) the Bag model equation of state with phase transition. For the RHLLE run we used a 2000 cell grid with $\Delta x = 0.02/\lambda$ fm, $\lambda = 0.99$. The larger prediffusion [29] for the SHASTA visible in (a) is expected on account of the smaller Courant–Friedrichs–Lewy number 0.4 as compared to 0.99 used in the RHLLE run [29]. The sightly slower cooling of the center is due to the larger numerical diffusion of the SHASTA. The sharp cusp-like structures that develop in the RHLLE solution in (b) and which are associated with deflagration discontinuities in the transition from quark–gluon to hadron matter [33, 34], are therefore also broadened in the SHASTA calculation. (The numerical diffusion for the RHLLE is so small that it even tends to produce small-scale oscillations around the true solutions at the origin and for late times in (b).) Up to these small numerical effects, however, agreement is satisfactory and establishes confidence in that the SHASTA algorithm is able to generate approximately correct hydrodynamical solutions also for more complicated initial conditions.

A third test of the numerical stability of the algorithm is shown in Fig. 7, comparing the experimentally observable transverse energy flow, $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$, and the azimuthal correlations of the transverse energy flow, $C_{ET}(\Delta\phi)$, in the case of ideal gas and Bag model equations of state. An initially azimuthally symmetric Gaussian energy density profile of radius 1 fm is evolved 100 time steps (with time step width $\Delta t = \lambda\,\Delta x = 0.08$ fm/c). The thermal smeared transverse energy distribution is computed as a function of time via eq. (29) derived in the Appendix. The azimuthal transverse energy correlation is defined by

$$C_{ET}(\Delta\phi) = \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, \frac{E_\perp(\phi+\Delta\phi)\; E_\perp(\phi)}{\langle E_\perp\rangle^2} - 1 \; , \tag{14}$$

where we abbreviated $E_\perp(\phi) \equiv \mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ and $\langle E_\perp\rangle \equiv \int \mathrm{d}\phi\, E_\perp(\phi)/2\pi$. For this test case, the exact solution has $E_\perp(\phi) = \langle E_\perp\rangle$ independent of ϕ and $C_{ET} = 0$. As shown in Fig. 7, while the initial condition agrees with that expectation, numerical errors develop at later times due to the cartesian grid that breaks the symmetry of the initial conditions. This is especially obvious from the fact that the azimuthal anisotropy develops directed along the four diagonal directions of the grid. The numerical anisotropy is largest for the first order transition case and reaches 10% in the $E_\perp(\phi)$ distribution while the numerical $E_\perp$ correlations remain below 0.1%. These results indicate the magnitude of numerical errors we must keep in mind in order to assess the significance of results for more complex geometries.

3.2 Transverse Shocks in Inhomogeneous Geometries

In this section we demonstrate that a novel class of azimuthally asymmetric collective flow patterns can develop from inhomogeneous initial conditions. These patterns are analogous to the hadronic volcanoes proposed by T.D. Lee [35]. However, instead of emerging from instabilities at the hadronization surface, these "volcanoes" arise from transverse directed shock waves formed during the expansion of initial state inhomogeneities [36].

To illustrate this type of collectivity, consider the evolution of two idealized Gaussian hot spots with radii 1 fm and with centroids separated by 4 fm in the transverse $x-$direction. The initial transverse flow velocity is assumed to vanish and only the Bjorken longitudinal flow velocity (2) is taken into account. The initial energy density of each Gaussian is assumed to peak at 30 GeV/fm^3 to simulate the hot spots seen in Figs. 2–4.

In Figure 8 the evolution of the expanding hot spots is shown for an ideal gas equation of state. The shaded energy density contours are shown every 10 computational time steps, corresponding to 0.8 fm/c between each frame. The expansion proceeds, as expected [37], with the formation of two (cylindrical) shells. At 2.9 fm/c (left column, middle row), a high density wall of shocked matter has formed at $x = 0$ where the expanding shells intersect. Subsequent evolution of that dense region leads to the "eruption" of two back-to-back jets of matter in the transverse $y-$direction. These transverse shock patterns are similar to the familiar "squeeze-out" flow pattern [38] produced in lower energy nuclear collisions. However, they are produced here by the collision of radially expanding shells or bubbles of relativistic hot matter, and in the case of multiple initial inhomogeneities, multiple volcanoes form at azimuthal angles that depend on the particular geometry.

These transverse shocks are most clearly visible in the $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ distribution as shown in Fig. 9. The initial $\mathrm{d}E_\perp(\tau_{th})/\mathrm{d}y\mathrm{d}\phi \simeq 24$ GeV is of course rotation invariant. However, at the end of the evolution ($\tau = 7.7$ fm/c), two narrow towers of directed transverse energy emerge at $\phi = 90$ and 270 degrees. This occurs because the initial azimuthal asymmetry in coordinate space is transferred into azimuthal asymmetry in momentum space through evolution. Finite dissipative effects would of course decrease the intensity and broaden the azimuthal distribution of these volcanoes.

Note also that the overall magnitude of the transverse energy decreases with time because work is performed by the fluid associated with longitudinal Bjorken expansion. The Gaussian hot spots expand not only in the transverse direction but also along the beam direction.

The initial $E_\perp$–correlation function is of course zero. By the time of freeze-out ($\mathcal{E} \sim 0.1$ GeV/fm^3), however, a strong forward and backward correlation develops for this geometry. It is important to observe that the magnitude of the correlation is only 30% even though the collective signal-to-thermal noise ratio, s, in $\mathrm{d}E_\perp(\tau_f = 7.7\,\mathrm{fm/c})/\mathrm{d}y\mathrm{d}\phi$ is peaked at $s \simeq 4$. This is a general feature of correlation functions since the convolution introduces a dependence on the width of the signal as well. If the relative width of the signal to noise is δ, then a simple estimate of the auto-correlation is

$$C(0) \simeq \frac{(s-1)^2\delta(1-\delta)}{(1+\delta(s-1))^2} \quad . \tag{15}$$

We can understand qualitatively the magnitude of the correlation at $\Delta\phi = 0$ in Fig. 9 taking

$s \simeq 3$ and $\delta \simeq 0.2$. The off-peak anti-correlation can also be estimated as

$$C(\pi/2) \simeq -\frac{(s-1)^2\delta^2}{(1+\delta(s-1))^2} \ , \tag{16}$$

which is also in qualitative accord with the computed correlation. These correlations are numerically significant because from Fig. 7 we found that the numerical errors lead only to a 0.01% correlation in the ideal gas case. The slight forward-backward asymmetry in the correlation function is only due to our histogram binning procedure.

In Figure 10 we show the evolution of the same initial condition assuming the Bag model equation of state, eq. (12). The expansion in this case is qualitatively different from the ideal gas case. The expanding shells or bubble walls are much thinner as is also the high density intersection region. This is even more clearly seen in Fig. 11 that shows the energy density as a function of time along the $x-$axis for both equations of state. The transverse velocity profile along that slice is also shown. In the case of a first order transition, a sharp cusp is produced at $x \simeq -5, 0, 5$ fm, which correspond to points where matter cools to the critical mixed phase transition point, ϵ_Q. In contrast, the bubbles and transverse shock zones in the ideal gas case remain comparable to the width of the initial hot spots. Those structures are much thinner in the phase transition case. The reason is that mixed phase matter does not resist compression due to the vanishing of the speed of sound and therefore fills less space than an ideal gas with finite compressibility (see also Fig. 6).

Also, the evolution in the case of a first order transition is slower by about a factor of two relative to Fig. 8. This is due to the stall of the expansion in the mixed phase because of the vanishing velocity of sound in those regions [8, 33]. In the ideal gas case, we have $c_s^2 = 1/3$ throughout the expansion. With $c_s = 0$ in the region with $\epsilon_H < \epsilon < \epsilon_Q$, sharp cusps are formed that move outwards only slowly. As can be seen from the velocity profiles the flow velocity has a discontinuity through the cusp, typical of deflagration phenomena [8, 33, 34].

The hydrodynamic stability analysis of bubble formation is complex and in the cosmological electroweak context is still subject to controversy [39, 40]. In the QCD case even less is known, though at least in [39] marginal stability was found to be possible within the uncertainties of the relevant scale. In our case, bubble formation does not result from supercooling but rather from the dynamical expansion of initial state inhomogeneities. Whether these hydrodynamic structures are stable is left here as an open question, especially since the thickness of the bubble walls is of hadronic dimensions. As we show in the next section the stalled expansion and the formation of thin shells of expanding mixed phase matter is the typical pattern we find also for the more complex inhomogeneous, turbulent mini-jet initial conditions.

Returning to Fig. 9, we see that the consequence of stalled expansion is a considerable reduction of the transverse shock intensity as measured by $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$, relative to the ideal gas case. The signal to noise is reduced to $s \simeq 1.5$ and the relative width is also reduced to $\delta \simeq 0.1$. With these reductions, the $E_\perp$ correlation is seen to be reduced by an order of magnitude in accord with eqs. (15,16). However, the few–percent correlation is still numerically significant compared to the 0.1% numerical errors deduced from Fig. 7.

3.3 Evolution of Turbulent Mini-Jet Initial Conditions

In Figures 12, 13 we compare the evolution of a typical mini-jet initial condition. The initial $T^{\mu\nu}$ required for the hydrodynamic evolution is taken from eq. (3) with $T^{00} \equiv \mathcal{E}$, $T^{0i} \equiv M^i$. We note that the HIJING initial conditions are not of the ideal fluid form. By only taking the left hand side of (3) to fix all other components of the fluid energy–momentum tensor, we convert the HIJING initial conditions into an ideal fluid form through the assumption of thermalization at τ_{th}. Thermalization has the effect of *reducing* the initial transverse energy somewhat from the HIJING input, because some of the transverse energy is converted into longitudinal thermal motion.

The time steps between frames in the case of a first order transition (Fig. 13) are taken to be twice as long (1.6 fm/c) as in the ideal gas case (Fig. 12) to take into account the inherently slower and stalled expansion in the first order transition case. For this event, several prominent hot spot regions are seen to expand in a manner analogous to the previous Gaussian examples. However, in this case the hot spots also start with initial transverse collective flow velocities determined from the mini-jet fluctuations.

The background fluid produced via soft beam-jet fragmentation in this example is assumed to have the smooth, azimuthally symmetric profile

$$\mathcal{E}^{\rm soft}(\tau_{th}, \mathbf{x}_\perp) = \frac{{\rm d}E_\perp^{\rm soft}}{{\rm d}y} \frac{1}{\tau_{th}\pi R^2} \frac{3}{2}(1 - r_\perp^2/R^2)^{1/2} \quad , \tag{17}$$

with $\mathbf{M}_\perp^{soft} = 0$ as well. Unlike in Figs. 2–4 where we included only about one half of the soft transverse energy on account of the formation time estimates, in this case we have taken the full ${\rm d}E_\perp^{\rm soft}/{\rm d}y = 0.5$ TeV estimated from HIJING. The soft component adds in this case (i.e. $p_0 = 2$ GeV/c) a larger smooth background of depth $\mathcal{E}^{\rm soft}(\tau_{th}, 0) \simeq 10$ GeV/fm^3 to the central region. As emphasized before, the magnitude of that soft background component is uncertain to about a factor two, and we take the above value to be on the conservative side.

In Figure 12 one can see two main expanding bubbles emerging from the two hottest spots. Their evolution is more irregular than in Fig. 8 because of the inhomogeneous background in which they propagate. The solid curve shows the freeze-out hadronization contour with $\epsilon = \epsilon_H = 0.1$ GeV/fm^3. In this ideal gas case, this hadronization surface shrinks monotonically but does not correspond to any obvious $\mathcal{E}$ contour because of the underlying chaotic collective flow velocity field.

In Figure 13, in addition to slowing down of the expansion, the evolution with the first order transition leads to the production of much thinner and irregular bubble fragments of mixed phase matter reminiscent of the thin shells in Fig. 10. It is the multiple inhomogeneities and shear velocity fields that make these structures much more irregular in this case. The hadronization surface ($\epsilon(\tau, \mathbf{x}_\perp) = \epsilon_H$) also seems to acquire a foam-like structure[‡]. The hadronization surface tends in this case of a first order transition to coincide closer with the plotted $\mathcal{E}(\tau, \mathbf{x}_\perp) = T^{00}(\tau, \mathbf{x}_\perp)$ contours because the velocity of the bubble walls are smaller than in the ideal gas case while the ejected cooler hadronic matter tends to have higher flow velocity (see [34]).

[‡]Bubble formation is a natural characteristic of a mixed phase, the above foam, however, is induced by the initial inhomogeneities and the subsequent collective motion.

In Figure 14, a cut along the $x-$axis provides a close-up of the complex nature of the sharp structures of mixed phase matter that emerge as well as of the chaotic transverse velocity fields in between them. It also shows the energy density scales corresponding to the shaded contours in Figs. 12 and 13. In this view, the sharp initial inhomogeneities due to mini-jets superimposed on the smooth beam-jet background and the initial turbulent velocity field are particularly clearly revealed.

The evolution of the transverse energy in this event is shown in Fig. 15. Unlike in the static examples discussed before, the initial $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ is not azimuthally symmetric in this case because of the initial turbulent velocity field. Of course, the azimuthal angles of the bumps and valleys vary from event to event. That is why correlation functions must be studied experimentally. However, the evolution of $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ in this event reveals the general tendency of inhomogeneous initial conditions to evolve into multiple azimuthally directed flow structures. Comparing the ideal gas and first order transition cases, we see again that the former leads to a lower average final transverse energy than the latter due to extra work done by the ideal gas upon longitudinal expansion.

In Figure 16 we show $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ averaged over 50 events and $C_{ET}(\Delta\phi)$ for such turbulent initial conditions. The event-averaged $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ remains approximately azimuthally symmetric as required. Initially, there is a small ($< 1\%$) azimuthal auto-correlation that is induced when the HIJING parton data are coarse-grained into 1 fm^3 fluid cells. The initial state correlation also includes the small fluctuating dipole contribution arising from the fact that $\sum \mathbf{p}_{\perp\alpha} \neq 0$ for a finite number of mini jets in the central rapidity slice. At later times, however, a 3% auto-correlation develops in the ideal fluid case and approximately 2% in the first order transition case. The magnitude of the correlations for both equations of state is evidently small.

In Figure 17, the dependence of the induced $E_\perp$ correlations on the soft background as well as on the mini-jet scale parameter p_0 is shown. The HIJING default case from Fig. 16 corresponds to the solid curves. In the ideal gas case (left panels), the auto-correlation reaches 6% if the background is reduced by a factor of two (dashed curve) to $\mathrm{d}E_\perp^{soft}/\mathrm{d}y = 250$ GeV as in Figs. 2–4. On the other hand, with $p_0 = 2$ GeV/c fixed but the soft background increased by a factor of two (dotted curve) to $\mathrm{d}E_\perp^{soft}/\mathrm{d}y = 1$ TeV, the final auto-correlation is reduced by around a factor of two relative to the default case. Finally, if p_0 is decreased to 1 GeV/c but $\mathrm{d}E_\perp^{soft}/\mathrm{d}y = 0.5$ TeV is kept fixed (dash–dotted curve), the correlation hardly changes relative to the corresponding $p_0 = 2$ GeV/c case (solid). The wiggle in this last case is due to the more limited statistics (20 events) available for this average. We conclude that in the ideal gas case, the collective azimuthal anisotropies are approximately linearly dependent on the level of the soft beam-jet component.

The initial state correlation function, however, also depends on the p_0 and soft background scales. In the lower panels, the ratio of the final correlation function to the initial one is shown in the restricted $\Delta\phi < \pi/2$ interval that avoids the artificial pole created at the point where the initial correlation function crosses zero. It is interesting to note that this ratio for the three $p_0 = 2$ GeV/c curves is practically independent of the background level. On the other hand, the $p_0 = 1$ GeV/c ratio is significantly larger. Thus, while the absolute magnitude of the correlation function depends roughly linearly on the soft background level, the dynamical enhancement of the initial state correlations in the ideal gas case peaks near

4–5 at $\Delta\phi = 0$ approximately independent of that background.

While the absolute collective signature as measured by the $E_\perp$–correlation function is small, it is quite significant compared to the initial state correlations (Fig. 16) and the numerical accuracy (Fig. 7). The transverse energy correlation function is of course only one way to search for azimuthally asymmetric collective flow phenomena. The power spectrum, $E_\perp(m) = \langle \int \mathrm{d}\phi \; e^{-im\phi} \, E_\perp(\phi) \rangle$, wavelet analysis, and factorial moment fluctuation analysis may provide more sensitive probes of the induced collectivity that is apparent in the ratio curves in Fig. 17.

In the case of the first order phase transition, the E_T correlations are significantly suppressed relative to the ideal gas case and appear to be much more insensitive to the background level. Comparing the solid and dotted curves indicates, however, a stronger dependence on the p_0 mini-jet scale. For $p_0 = 1$ GeV/c and $\mathrm{d}E_\perp^{soft}/\mathrm{d}y = 1$ TeV, the azimuthal anisotropies fall below 1%. On the other hand, in the ratio of final to initial correlation functions, the largest enhancement occurs for the $p_0 = 1$ GeV/c case. The ratios also show a qualitative shoulder feature in the first order transition case for which we have not found a simple explanation. The level of collectivity is, however, probably too small to see such structures.

The suppression of collective flow phenomena in the case of a first order transition is due to the vanishing of the speed of sound over a large interval of energy densities, $\epsilon_H < \epsilon < \epsilon_Q$. This is also manifest in the smaller reduction of the initial $\mathrm{d}E_\perp/\mathrm{d}y$ due to longitudinal expansion relative to the expansion with an ideal gas equation of state. The above results can be understood as a consequence of a rather general feature of evolution with any equation of state that possesses a dip of $c_s^2 = \mathrm{d}p/\mathrm{d}\epsilon$ in a finite range of energy densities. In the case of a first order transition, $c_s^2 = 0$ in the mixed phase, and pressure gradients driving collective flow phenomena are strongly suppressed. As emphasized in [8], even a continuous cross-over transition may feature such a minimum if the cross-over temperature region is not too broad. For realistic $\Delta T/T_c \simeq 0.1$, the softening of the equation of state is sufficiently strong that the time-delay signature discussed in the next section should still be observable. The same physics of softening is expected to lead to a suppression of directed transverse flow phenomena at much lower AGS energies [41] and also to the suppression [42] of $\mathrm{d}\langle p_\perp \rangle / \mathrm{d}(\mathrm{d}N_\pi/\mathrm{d}y)$ in the mixed phase region. In the turbulent-glue scenario, the suppression of pressure gradients in the cross-over region of the equation of state has the observable consequence of reducing the azimuthally asymmetric collective phenomena.

It is indeed curious that many "barometric" signatures of QGP formation involve the *suppression* of collective behaviour that would otherwise naturally arise in ideal gas systems. This makes the search for signatures of the QGP more difficult because ordinary dissipative effects due to viscocity, thermal and colour conductivity etc. work in the same direction. It is only through the careful systematic studies of all observables as a function of beam energy and nuclear size that one can hope to unravel interesting threshold-type behaviour caused by the passage through a mixed-phase region from the suppression of collective phenomena due to less interesting dissipative dynamics.

4 Robustness of the Time-Delay Signature

Since it is the reduction of collective observables in the evolution of a QGP that signals rapid cross-over regions in the equation of state, it is important to find as many correlated signatures as possible in the search for that phase of matter. As repeatedly stressed above, one of the generic consequences of hydrodynamics with an equation of state that has a soft region (reduction of c_s^2) is time delay. Meson interferometry has been proposed [6, 7] as the main experimental tool to search for such time-delay signatures. As shown recently in [8], that signature of stalled dynamics is fortunately robust to an increase in the width of the cross-over region. In this section, we want to demonstrate in more detail the robust character of that observable even to the much more unfavourable turbulent initial conditions discussed above.

In Figure 18, the evolution of the *mean* energy density is shown, averaged over the inner $r_\perp < 3$ fm core of the plasma. The average is again over 50 events for each equation of state. At $\tau = \tau_{th} = 0.5$ fm/c, the mean central energy density is approximately 16 GeV/fm^3 for the turbulent ensemble with $p_0 = 2$ GeV/c and $\mathrm{d}E_\perp^{soft}/\mathrm{d}y = 0.5$ TeV. The solid curve, for the case of a first order transition and 3+1–dimensional expansion, should be compared to the thick dashed curve for the case of an ideal gas equation of state. In addition, the light dashed and dash–dotted curves are shown for comparison. They correspond to ideal one–dimensional Bjorken expansion with transverse expansion neglected. The τ^{-1} curve represents pure longitudinal expansion without work, $p = 0$, while the $\tau^{-4/3}$ curve corresponds to the ideal gas case. The 3+1–dimensional ideal gas evolution starts to deviate from the one–dimensional case after a short time ~ 2 fm/c due to rapid radial expansion. In the first order transition case, the mean energy density follows the ideal one–dimensional Bjorken curve up to ~ 6 fm/c because the transverse expansion is stalled. The freeze-out occurs near $\epsilon \simeq \epsilon_H \simeq 0.1$ GeV/fm^3. It is clear from Fig. 18 that the freeze-out time is approximately twice as long in the case of a phase transition to the QGP.

Especially important for the $R_{\mathrm{out}}/R_{\mathrm{side}}$ signature [6, 7, 8] of the QGP is that the transverse coordinate distribution in the first order transition case remains more compact even though it takes a longer time for the system to freeze-out. This can be seen clearly in Figs. 12 and 13 which, together with Fig. 18, confirm that the space–time geometry of freeze-out remains so different in the two cases.

Figure 19 emphasizes the strongly inhomogeneous character of expansion in the turbulent-glue scenario. The thick solid and dashed curves are the same as in Fig. 18. The two thin curves show the magnitude of the rms fluctuations of the energy density in the central core for the first order transition case. The initial fluctuations are already large. However, those fluctuations grow rapidly as the system passes through the mixed phase and the foam structure in Fig. 13 develops. This shows that freeze-out is not a fixed-time event, but is spread over the entire evolution. The most essential aspect for the time-delay signal is that the freeze-out time duration is large relative to the transverse coordinate dispersion [6, 7, 8].

The actual computation of the pion interference pattern from the turbulent evolution is beyond the scope of the present study. A substantial generalization of the already computationally demanding methods used in Ref. [8] will be required. However, based on that work, we expect the general interference pattern to reflect well the underlying, time-delayed freeze-

out geometry. Therefore, the time-delay signature survives even if the initial conditions are as inhomogeneous as in the turbulent-glue scenario.

The detailed interference pattern can be expected, on the other hand, to differ from the patterns produced from the evolution of homogeneous initial conditions due to the overlapping bubble wall geometries found in Fig. 13. The Fourier transform of such hadronic foam is bound to contain extra structure since multiple distance scales (shell thickness and radii, and relative separations) enter the coordinate space distribution. It would be interesting in the future to explore such novel interference patterns that would be specific to inhomogeneous geometries.

5 Enhanced Radiance of Hot Spots

A specific consequence of hot-spot formation is enhanced radiance of hard probes. For instance, the (invariant) rate for emitting photons with momentum $\mathbf{k}$ from a fluid element consisting of quarks and gluons with temperature T and 4–velocity u^μ is given by [43]

$$\omega \frac{\mathrm{d}R^\gamma}{\mathrm{d}^3\mathbf{k}} = \frac{5\alpha\alpha_S}{18\pi^2} T^2 e^{-k\cdot u/T} \ln\left(\frac{2.912\, k\cdot u}{g^2 T} + 1\right) \quad . \tag{18}$$

It was moreover shown in [43] that this rate is approximately the same if the fluid element consists of hadrons instead of QGP. To estimate photon radiation in the turbulent-glue scenario as compared to that from a homogeneous Bjorken cylinder, we integrate numerically the rate (18) over the photon emission angle, over (proper) time and transverse coordinates, and evaluate it at $y = \eta = 0$ ($\eta = \ln[(t+z)/(t-z)]/2$ being the space–time rapidity).

The final differential photon spectrum $\mathrm{d}N^\gamma/\mathrm{d}\eta\mathrm{d}yk_\perp\mathrm{d}k_\perp|_{y=\eta=0}$ is shown in Fig. 20 averaged over 10 HIJING events (thick lines) and for a homogeneous Bjorken cylinder (thin lines) of radius $R = 4$ fm with the same initial (average) energy as the HIJING events. We note that even a single event produces a quite similar spectrum as the 10–event average. As one expects, the higher temperatures of hot spots in the turbulent-glue scenario lead to an enhancement of the exponential tail of the photon spectrum.

Comparing the yield in the case of a phase transition (dashed lines) to that in the case of an ideal gas equation of state (solid lines), one observes that the longer lifetime of the system in the case of a phase transition leads to enhanced radiation of photons with small as well as with large momenta in the case of the turbulent-glue scenario, while for the homogeneous Bjorken cylinder, only radiation of photons with small momenta is significantly enhanced. The higher fluid velocities in the case of the first order transition scenario as well as reheating in the shocks created in the expansion of hot spots seem to be responsible for the stronger population of the high-momentum tail of the spectrum. Correlations of direct gammas with azimuthally directed transverse shocks should also be looked for. Other probes such as strangeness enhancement may also be correlated with transverse shocks.

6 Summary

In this paper we studied several possible consequences of initial state inhomogeneities and turbulence in quark–gluon plasmas arising from multiple mini-jet production in ultrarelativistic nuclear collisions. The HIJING model was used to generate the initial mini-jet configurations and to provide an estimate of the soft beam-jet background. The ensemble of initial conditions was found to exhibit a wide spectrum of fluctuations not only of the initial energy density distribution but also of the initial transverse velocity field. The fluctuations are large if the equilibration time of the plasma is short, ~ 0.5 fm/c, as suggested by recent estimates of radiation energy loss. We refer to this type of initial conditions as the turbulent-glue scenario to contrast it with the more conventional hot-glue scenario which assumes cylindrical symmetry, homogeneity, and quiescence of the initial plasma.

We assumed the validity of non-dissipative hydrodynamics in order to assess the most optimistic observable consequences of such unfavourable initial conditions. We studied three observables that could serve as diagnostic tools of such plasmas.

First, we showed that new types of azimuthally asymmetric collective flow patterns (volcanoes) could arise in inhomogeneous geometries. At the event-by-event level they could be observed by looking for spikes in the transverse energy distribution, $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$. However, the collective effects are difficult to identify because of large uncorrelated fluctuations associated with the turbulent nature of the initial conditions. Those uncorrelated fluctuations can be averaged out by studying instead the azimuthal correlation function of the transverse energy. The remaining dynamical correlations are small but sensitive to the plasma equation of state. In general, evolution with an equation of state featuring a minimum of the speed of sound in the energy density range of interest reduces all collective flow phenomena relative to the ideal gas case. For example, the overall reduction of the initial transverse energy due to work associated with longitudinal expansion is maximal for the ideal gas case.

Second, we discussed the time-delay signature of a QGP transition. We found that, as in the conventional hot-glue scenario, the evolution is stalled if there is a soft region of the equation of state. It is remarkable that this phenomenon survives not only if the QGP equation of state only features a smooth cross-over instead of a first order transition [8, 33], but also if the ensemble of initial conditions is as complex as in the turbulent glue-scenario. Meson interferometry [6, 7, 8] appears therefore to be one of the most robust diagnostic tools in the search for the QGP transition.

Finally, we considered briefly the effect of hot spots on hard probes. The thermal contribution to probes such as direct photons is obviously exponentially sensitive to the local temperature. We showed that the turbulent-glue scenario can enhance by more than an order of magnitude the high–$k_\perp$ tails for both equations of state. Other hard probes and even softer ones such as strangeness production can be expected to be correlated and enhanced due to hot spots and collective transverse shock phenomena. However, to identify any enhanced yields with hot-spot formation will require subtraction of other pre-equilibrium contributions to those yields.

The above scenario represents in many ways the most optimistic point of view for signatures of QGP formation. We neglected all dissipative effects that tend to dampen collective behaviour of the system and decrease the freeze-out time. The thin expansion shell

structures found may be diffused considerably by such effects. Chemical equilibrium may not be maintained through the transition point. In general, chemical rate equations would have to supplement the hydrodynamic equations. Also, we assumed Bjorken boundary conditions and hence neglected fluctuations along the longitudinal direction. In the general 3+1–dimensional case, fluctuations will produce hot-spot droplets instead of the cylindrical structures studied here. That extra dimension decreases the overlap phase space of expanding inhomogeneities and therefore reduces the pattern of collective transverse shocks. We have also assumed a most conservative homogeneous beam-jet background underneath the turbulent mini-jet plasma. In confronting data eventually, these and other complications will have to be considered in more detail in future studies.

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Appendix: The Transverse Energy Flow

The local thermal phase space density for a mixture of massless fermions and bosons is

$$f(t,\mathbf{x},\mathbf{p}) = \frac{\mathrm{d}^6 N}{\mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{p}} = \frac{1}{(2\pi)^3}\left(\frac{g_B}{e^{p\cdot u/T}-1} + \frac{g_F}{e^{p\cdot u/T}+1}\right) \equiv h\left(\frac{p\cdot u}{T}\right) \quad , \tag{19}$$

where $T(x)$ and $u^\mu(x)$ are the local temperature and fluid 4–velocity, and $g_B(g_F)$ are the number of active massless Boson (Fermion) helicity states. For example, in the mixed phase $g_B = 16r_Q + 3(1-r_Q)$ if the fraction of matter in the plasma phase is r_Q. The fluid energy–momentum tensor is related to f via

$$T^{\mu\nu}(x) = \int \frac{\mathrm{d}^3\mathbf{p}}{p^0}\, p^\mu p^\nu h\left(\frac{p\cdot u}{T}\right) \quad . \tag{20}$$

For longitudinal boost-invariant (Bjorken) boundary conditions, $T = T(\tau, \mathbf{x}_\perp)$, depends only on the transverse coordinates $\mathbf{x}_\perp$ and the proper time $\tau = \sqrt{t^2 - z^2}$. The longitudinal fluid velocity is determined by the space–time rapidity variable $\eta = \ln[(t+z)/(t-z)]/2$, via $v^z = u^z/u^0 = \tanh\eta$. If the transverse flow velocity at $z = 0$ ($\eta = v^z = 0$) is specified as $\mathbf{v}_\perp = \tanh y_\perp(\cos\phi_\perp, \sin\phi_\perp)$ in terms of the local transverse flow rapidity $y_\perp(\tau, \mathbf{x}_\perp)$ and its azimuthal direction by the angle $\phi_\perp(\tau, \mathbf{x}_\perp)$, then the fluid 4–velocity is given by

$$\begin{aligned} u^\mu(\tau, \mathbf{x}_\perp, \eta) &= (\cosh\eta\cosh y_\perp, \sinh y_\perp\cos\phi_\perp, \sinh y_\perp \sin\phi_\perp, \sinh\eta\cosh y_\perp) \\ &= \gamma_\perp(\cosh\eta, v_\perp\cos\phi_\perp, v_\perp\sin\phi_\perp, \sinh\eta) \quad , \end{aligned} \tag{21}$$

where $\gamma_\perp \equiv \cosh y_\perp$ and $v_\perp \equiv \tanh y_\perp$. Note that if the transverse flow velocity vanishes, $y_\perp = 0$, then we recover the familiar Bjorken flow velocity field $u^\mu = x^\mu/\tau$.

It is convenient to express p^μ in terms of longitudinal rapidity and cylindrical transverse momentum variables, such that $p^\mu = p_\perp(\cosh y, \cos\phi, \sin\phi, \sinh y)$. Therefore, the argument of the thermal distribution h above reduces to

$$p \cdot u/T = p_\perp \gamma_\perp (\cosh(y-\eta) - v_\perp \cos(\phi - \phi_\perp))/T \tag{22}$$

It is linearly dependent on $p_\perp$ because we assume that all components of the fluid are massless.

The energy density at $z = 0$, i.e. $\eta = 0$, is then given by $\mathcal{E} = T^{00}$ via

$$\mathcal{E}(\tau, \mathbf{x}_\perp, 0) = \left(\frac{T}{\gamma_\perp}\right)^4 \int_{-\infty}^{\infty} \mathrm{d}y\, \cosh^2 y \int_0^{2\pi} \frac{\mathrm{d}\phi}{(\cosh y - v_\perp \cos\phi)^4} \int_0^\infty \mathrm{d}x x^3 h(x) \quad . \tag{23}$$

Note that

$$\int_0^\infty \mathrm{d}x x^3 h(x) = \frac{1}{4\pi}(g_B + 7g_F/8)\frac{\pi^2}{30} \quad . \tag{24}$$

When this last factor is combined with T^4 we obtain the local *proper* energy density $\epsilon(\tau, \mathbf{x}_\perp)$ $/(4\pi)$. In the limit where $v_\perp(\tau, \mathbf{x}_\perp) = 0$, the remaining two integrals then yield a factor 4π and thus eq. (23) reduces to the expected

$$\lim_{v_\perp \to 0} \mathcal{E}(\tau, \mathbf{x}_\perp, 0) = (g_B + 7g_F/8)\, \frac{\pi^2}{30}\, T^4(\tau, \mathbf{x}_\perp) \equiv \epsilon(\tau, \mathbf{x}_\perp) \quad . \tag{25}$$

When the local transverse flow velocity does not vanish, the last two integrals are still analytic and implement the correct Lorentz transformation of the energy density in the ideal fluid limit with

$$\mathcal{E}(\tau, \mathbf{x}_\perp, 0) = \frac{1}{3}\,(4\gamma_\perp^2 - 1)\, \epsilon(\tau, \mathbf{x}_\perp) \quad . \tag{26}$$

(Recall that for an ideal fluid $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p g^{\mu\nu}$.)

If the fluid is frozen out at a fixed proper time, τ_f, then the invariant total particle distribution is obtained by integrating over all fluid cells via the Cooper–Frye formula

$$\begin{aligned} p\,\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} &= \frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}_\perp} = \int_{\Sigma_f} \mathrm{d}\sigma^\mu\, p_\mu\, f(\tau_f, \mathbf{x}, \mathbf{p}) \\ &= \tau_f p_\perp \int \mathrm{d}^2\mathbf{x}_\perp \int \mathrm{d}\eta\, \cosh(\eta - y)\, h\left(\frac{p \cdot u}{T}\right) \quad , \end{aligned} \tag{27}$$

where we used the 3–volume element

$$\mathrm{d}\sigma^\mu = \epsilon^{\mu\alpha\beta\gamma}\, \frac{\partial x_\alpha}{\partial \eta} \frac{\partial x_\beta}{\partial x_\perp} \frac{\partial x_\gamma}{\partial y_\perp}\, \mathrm{d}\eta\, \mathrm{d}^2\mathbf{x}_\perp = (\cosh\eta, 0, 0, \sinh\eta)\, \tau_f\, \mathrm{d}\eta\, \mathrm{d}^2\mathbf{x}_\perp \tag{28}$$

given $x^\mu(\tau, \mathbf{x}_\perp, \eta) = (\tau\cosh\eta, \mathbf{x}_\perp, \tau\sinh\eta)$. Note that $\mathrm{d}N/\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp$ is of course independent of y given our boundary conditions, but if $y_\perp(\tau, \mathbf{x}_\perp) \neq 0$ it could be highly azimuthally asymmetric.

Finally, the transverse energy distribution at proper time τ_f and $y = 0$ is given by

$$\begin{aligned}\frac{\mathrm{d}E_\perp}{\mathrm{d}y\mathrm{d}\phi} &= \int_0^\infty \mathrm{d}p_\perp\, p_\perp^2 \frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \tau_f \int \mathrm{d}^2\mathbf{x}_\perp \int \mathrm{d}\eta\ \cosh\eta \int \mathrm{d}p_\perp\, p_\perp^3 h\left(\frac{p\cdot u}{T}\right) \\ &= \tau_f \int \mathrm{d}^2\mathbf{x}_\perp\ \frac{\epsilon(\tau_f,\mathbf{x}_\perp)}{4\pi\gamma_\perp^4} \int \mathrm{d}\eta \frac{\cosh\eta}{(\cosh\eta - v_\perp\cos(\phi-\phi_\perp))^4} \\ &= \int \mathrm{d}^2\mathbf{x}_\perp\, \tau_f \mathcal{E}(\tau_f,\mathbf{x}_\perp)\ \Delta(\phi - \phi_\perp(\tau_f,\mathbf{x}_\perp), v_\perp(\tau_f,\mathbf{x}_\perp))\ ,\end{aligned} \tag{29}$$

where the smeared-out azimuthal angle delta function is given by

$$\begin{aligned}\Delta(\phi,v) &= \frac{3}{4\pi\gamma^4(4\gamma^2-1)} \int_{-\infty}^{\infty} \mathrm{d}\eta\ \frac{\cosh\eta}{(\cosh\eta - v\ \cos\phi)^4} \\ &= \frac{1}{4\pi}\frac{(1-v^2)^3}{3+v^2}\left\{\frac{w(2w^2+13)}{(1-w^2)^3} + \frac{6+24w^2}{(1-w^2)^{7/2}}\arctan\sqrt{\frac{1+w}{1-w}}\right\}\ ,\end{aligned} \tag{30}$$

where $w = v\cos\phi$. In the $v = 0$ limit, $\Delta(\phi, 0) = 1/8$ which is $\pi/4$ times smaller than the naive $1/(2\pi)$ uniform distribution expected in the non-thermal Bjorken limit. In the $v \to 1$ limit, we can use the integral [44]

$$\int_0^{2\pi} \frac{\mathrm{d}\phi}{(a - b\cos\phi)^4} = \pi\frac{a(2a^2+3b^2)}{(a^2-b^2)^{7/2}} \tag{31}$$

to show that $\int \mathrm{d}\phi\, \Delta(\phi, v \to 1) = 1$, i.e.,

$$\lim_{v\to 1}\Delta(\phi,v) = \delta(\phi)\ . \tag{32}$$

References

[1] K. Kajantie, P.V. Landshoff, J. Lindfors, Phys. Rev. Lett. 59 (1987) 2527,
J.P. Blaizot, A.H. Mueller, Nucl. Phys. B 289 (1987) 847,
K.J. Eskola, K. Kajantie, J. Lindfors, Nucl. Phys. B 323 (1989) 37.

[2] E. Shuryak, Phys. Rev. Lett. 68 (1992) 3270.

[3] K.J. Eskola and M. Gyulassy, Phys. Rev. C 47 (1993) 2329.

[4] N.H. Christ, Nucl. Phys. A 544 (1992) 81c,
T. Blum, L. Kärkkäinen, D. Toussaint, S. Gottlieb, Phys. Rev. D 51 (1995) 5153.

[5] Quark Matter '93, eds. E. Stenlund, H.A. Gustafsson, A. Oskarsson, I. Otterlund, Nucl. Phys. A 566 (1994) 1c,
Quark Matter '95, eds. A. Poskanzer, J. Harris, L. Schroeder, Nucl. Phys. A 590 (1995) 1c.

[6] S. Pratt, Phys. Rev. D 33 (1986) 1314, Phys. Rev. C 49 (1994) 2722.

[7] G. Bertsch, M. Gong, M. Tohyama, Phys. Rev. C 37 (1988) 1896,
G. Bertsch, Nucl. Phys. A 498 (1989) 173c.

[8] Dirk H. Rischke and Miklos Gyulassy, Columbia preprint CU–TP–756 (1996), (http://xxx.lanl.gov/ps/nucl-th/9606039), Nucl. Phys. A (in press).

[9] X.N. Wang and M. Gyulassy, Phys. Rev. D 44 (1991) 3501, Phys. Rev. D 45 (1992) 844, Phys. Rev. Lett. 68 (1992) 1480, Phys. Lett. B 282 (1992) 466, Comp. Phys. Comm. 83 (1994) 307.

[10] T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74, CERN-TH-7112-93, Feb. 1994, unpublished, available on WWW internet at http://wwwcn.cern.ch/asdoc/Welcome.html.

[11] B. Andersson, G. Gustafson, B. Nilsson–Almqvist, Nucl. Phys. B 281 (1987) 289,
B. Nilsson–Almqvist, E. Stenlund, Comp. Phys. Commun. 43 (1987) 387,
A. Capella, U. Sukhatme, C.I. Tan, J. Tran Thanh Van, Phys. Rep. 236 (1994) 225.

[12] K. Geiger and B. Müller, Nucl. Phys. B 369 (1992) 600,
K. Geiger, Phys. Rep. 258 (1995) 237.

[13] L. McLerran and R. Venugopalan, Phys. Rev. D 49 (94) 2233, 3352, Phys. Rev. D 50 (94) 2225.

[14] H. Ehtamo, J. Lindfors, L. McLerran, Z. Phys. C 18 (1983) 341.

[15] M. Gyulassy and A. Iwazaki, Phys. Lett. B 165 (1985) 157.

[16] T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B 245 (1984) 449.

[17] L. Xiong and E.V. Shuryak, Proc. of the Workshop on Pre-Equilibrium Parton Dynamics, Berkeley, California, Aug. 23 – Sept. 3, 1993, p. 73.

[18] M. Gyulassy and X.N. Wang, Nucl. Phys. B 420 (1994) 583,
X.N. Wang, M. Gyulassy, M. Plümer, Phys. Rev. D 51 (1995) 3436.

[19] R. Baier, Yu.L. Dokshitzer, S. Peigné, D. Schiff, Phys. Lett. B 345 (1995) 277.

[20] J.D. Bjorken, Phys. Rev. D 27 (1983) 140.

[21] P. Danielewicz and M. Gyulassy, Phys. Rev. D 31 (1985) 53.

[22] E.V. Shuryak, Phys. Lett. B 78 (1978) 150.

[23] J.P. Blaizot and J.Y. Ollitrault, Phys. Rev. D 36 (1987) 916,
M. Kataja, J. Letessier, P.V. Ruuskanen, A. Tounsi, Z. Phys. C 55 (1992) 153.

[24] Enhanced fluctuations of $E_\perp$ can also be accounted assuming substantial fluctuations of hadronic cross sections: B. Blättel, G. Baym, L.L. Frankfurt, H. Heiselberg, M. Strikman, Phys. Rev. D 47 (1993) 2761.

[25] M.H. Thoma, Phys. Lett. B 273 (1991) 128,
A.V. Selikhov and M. Gyulassy, Phys. Rev. C 49 (1994) 1726.

[26] J.Y. Ollitrault, Phys. Rev. D 46 (1992) 229, Phys. Rev. D 48 (1993) 1132.

[27] G.A. Sod, J. Fluid Mech. 83 (1977) 785.

[28] J.P. Boris, D.L. Book, J. Comput. Phys. 11 (1973) 38,
D.L. Book, J.P. Boris, K. Hain, J. Comput. Phys. 18 (1975) 248.

[29] D.H. Rischke, S. Bernard, J.A. Maruhn, Nucl. Phys. A 595 (1995) 346.

[30] S. Zalesak, J. Comput. Phys. 31 (1979) 335.

[31] D.H. Rischke, Y. Pürsün, J.A. Maruhn, Nucl. Phys. A 595 (1995) 383.

[32] V. Schneider et al., J. Comput. Phys. 105 (1993) 92.

[33] D.H. Rischke and M. Gyulassy, Nucl. Phys. A 597 (1996) 701.

[34] M. Gyulassy, K. Kajantie, H. Kurki–Suonio, L. McLerran, Nucl. Phys. B 237 (1984) 477.

[35] T.D. Lee, Columbia preprint CU–TP–226 (1982), Proc. of DPF Summer Study on Elementary Particle Physics and Future Facilities, Snowmass, Colorado, 1982 (eds. R. Donaldson, R. Gustafson, F. Paige, Amer. Inst. Phys., 1983).

[36] M. Gyulassy, D.H. Rischke, B. Zhang, Columbia preprint CU–TP–757, (http://xxx.lanl.gov/ps/nucl-th/9606045), Proc. Int. Conf. on Nucl. Phys. at the Turn of Millennium, Wilderness/George, South Africa, March 1996 (in press).

[37] C. Greiner, D.H. Rischke, Columbia preprint CU–TP–745, (http://xxx.lanl.gov/ps/nucl-th/9604044) Phys. Rev. C (in press).

[38] H. Stöcker and W. Greiner, Phys. Rep. 137 (1986) 277.

[39] P. Huet, K. Kajantie, R.G. Leigh, B.-H. Liu, L. McLerran, Phys. Rev. D 48 (1993) 2477.

[40] M. Kamionkowski and K. Freese, Phys. Rev. Lett. 69 (1992) 2743.

[41] D.H. Rischke, Y. Pürsün, J.A. Maruhn, H. Stöcker, W. Greiner, Columbia preprint CU–TP–695, (http://xxx.lanl.gov/ps/nucl–th/9505014), to be published in Heavy Ion Phys.

[42] E.V. Shuryak and O.V. Zhirov, Phys. Lett. B 89 (1980) 253,
L. van Hove, Phys. Lett. B 118 (1982) 138,
M. Gyulassy, Nucl. Phys. A 418 (1984) 59c.

[43] J. Kapusta, P. Lichard, D. Seibert, Phys. Rev. D 44 (1991) 2774.

[44] I.S. Gradsteyn and I.M. Ryzhik, "Table of Integrals, Series, and Products", Academic Press, San Diego, 1980, eq. (3.661.4).

Figure Captions:

Fig. 1: HIJING1.3 model [9] predictions for central ($b = 0$) $Au + Au$ collisions at $\sqrt{s} = 200$ AGeV are shown for the mini-jet cut-off scale $p_0 = 1$ GeV/c (solid) and 2 GeV/c (dashed). The rapidity density of mini-jet gluons including initial and final state radiation is shown in part (a). Shadowing and jet quenching are not included in these calculations. The gluon transverse energy per unit rapidity is shown in part (b). In part (c) the initial transverse momentum distribution of the mini-jet gluons at $y = 0$ is shown. The final hadronic transverse energy distribution including both mini-jet and beam-jet fragmentation is shown in (d).

Fig. 2: (a) Hot spots in $Au + Au$ ($\sqrt{s} = 200$ AGeV, $b = 0$) at the thermalization time $t = \tau_{th} = 0.5$ fm/c are seen as spikes in the local energy density, $\mathcal{E}(\tau_{th}, \mathbf{x}_\perp)$, as a function of the transverse coordinate, $\mathbf{x}_\perp$, in the $z = 0$ plane. The transverse and longitudinal resolution scales are fixed to be $\Delta r_\perp = 1$ fm and $\Delta y = 1$. The mini-jet scale here is $p_0 = 2$ GeV/c. (b) The chaotic initial transverse momentum field, $\mathbf{M}_\perp(\tau_{th}, \mathbf{x}_\perp)$, is represented here by arrows. (c) The spectrum of proper energy density fluctuations at τ_{th} in the central region with $r_\perp < 4$ fm is shown. The histogram is an average of 200 HIJING events and includes a soft gluon component due to beam-jet fragmentation. (d) The spectrum of effective gluon temperatures in the central region corresponding to (c) is shown.

Fig. 3 : A surface plot of the local energy density $\mathcal{E}(\tau_{th}, \mathbf{x}_\perp)$, averaged over 200 events, shows the smooth, azimuthally symmetric form usually assumed in the hot-glue scenario. The shaded contour on the top part is a 2–dimensional projection of that surface plot. In parts (b)–(d), three separate HIJING events show that the fluctuations in Fig. 2a are typical. The shaded contours illustrate the complex nature of the azimuthally asymmetric inhomogeneities produced by the mini-jet mechanism.

Fig. 4 : The gluon number densities corresponding to the same events as in Fig. 3 show that the hot spots in Fig. 3 are not caused by isolated jets but occur as accidental coincidence of several gluons in the same region.

Fig. 5: Evolution of a cylindrically symmetric Gaussian hotspot of radius 2 fm and peak energy density $\mathcal{E} = 30$ GeV/fm^3. Shown are (calculational frame) energy density profiles for (a) the initial situation at time $t_0 = 0.5$ fm/c, (b) at time $t = 14.9$ fm/c after evolving with the standard antidiffusion fluxes and with the ideal gas equation of state, (d) at the same time after evolving with antidiffusion fluxes reduced to 70% of the standard value, and (c) at the same time after an evolution with the equation of state with a phase transition and reduced antidiffusion. (b)–(d) show the profiles only for positive x, y, the other quadrants are symmetric.

Fig. 6: (a) Energy density profiles along the $x-$axis for times $t = \tau_0 + 1.6\, n$ fm/c, $\tau_0 = 0.5$ fm/c, $n = 0, 1, ..., 9$, calculated with the ideal gas equation of state. Full lines are results from the RHLLE, dots are cell values for the SHASTA run. (b) Energy density profiles for times $t = \tau_0 + 3.2\, n$ fm/c, $n = 0, 1, .., 4$ and $t = 14.9$ fm/c, calculated with the equation of

state with a phase transition.

Fig. 7 : The evolution of numerical errors due to the cartesian grid is shown for the expansion of a single Gaussian initial condition. The transverse energy fluctuations are initially azimuthally symmetric but develop 10 % systematic asymmetries in the case of a first order transition (upper right panel). The maximal $E_\perp$ correlation induced by the grid is, however, only on the order of 0.1% (lower right panel). In the ideal gas case, errors are about an order of magnitude smaller (left panels) at the same time.

Fig. 8 : The evolution of the energy density contours from the expansion of two adjacent Gaussian hot spots is shown for the case of an ideal gas equation of state. Note the development of two back-to-back transverse jets from the shock zone. The time step between different frames is 0.8 fm/c, the initial time is 0.5 fm/c.

Fig. 9 : Azimuthally asymmetric transverse shocks produced during the evolution of the inhomogeneous initial condition in Fig. 8 are evident as spikes in the transverse energy distribution (upper panels) as well as enhanced $E_\perp$ correlations (lower panels). The initial $\mathrm{d}E_\perp/\mathrm{d}y\mathrm{d}\phi$ and C_{ET} are straight lines, due to the absence of collective motion in the initial conditions. Note that a first order transition (right panels) inhibits the formation of such transverse shocks because the expansion is stalled in that case (see Fig. 10). In both cases, the dynamically produced asymmetries are much larger than the numerical errors from Fig. 7.

Fig. 10: The evolution of the same initial condition as in Fig. 8 but assuming here the Bag model equation of state. The stalled expansion due to the vanishing of the speed of sound is revealed using larger time steps (1.6 fm/c) between frames relative to Fig. 8. Note that the expansion bubbles are much thinner as is the shock zone.

Fig. 11: The evolution of the energy density $\mathcal{E}$ along the $x-$axis of Figs. 8 and 9 shows clearly the difference between shell structures formed in the ideal gas vs. the first order transition cases. Lower graphs show the evolution of the $v^x(x, 0)$ transverse velocity field that vanishes initially. The time steps shown for the ideal case are $\tau = 0.5, 2.1, 3.7, 5.3, 7.7$ fm/c, while in the first order transition case they are $\tau = 0.5, 3.7, 6.9, 10.1, 14.9$ fm/c.

Fig. 12: The evolution of the energy density $\mathcal{E}$ for a typical HIJING event assuming an ideal gas equation state. The (solid) contour indicates the hadronization surface, where the proper energy density has the value $\epsilon_H \simeq 0.1$ GeV/fm^3. The time steps are indicated on the top right of each frame. Darker shaded regions correspond to hot spots while lighter shaded regions to cooler domains. See Fig. 14 below for a slice along the $x-$axis through these profiles that indicates the energy density scale.

Fig. 13: The evolution of the same HIJING event as in Fig. 12, but with the equation of state with a first order transition. Note that the time steps are twice as large as in Fig. 12 due to the stalled expansion with this equation of state. Also note the formation of thin bubble walls of mixed phase matter as in Fig. 10. The contour of $\epsilon = \epsilon_H$ shows that hadronic

foam-like structures develop due to the initial state inhomogeneities.

Fig. 14: A slice along the $x-$axis through selected frames in Figs. 12 and 13 indicates the magnitude of the energy densities in the structures formed during evolution. The $v^x(x,0)$ collective flow velocity field does not vanish at τ_{th} as in Fig. 11 because of the initial turbulence induced by mini-jet formation. The curves are plotted at the same times as in Fig. 11.

Fig. 15: The evolution of the thermal smeared transverse energy distribution corresponding to the event in Figs. 12 and 13. Note that the initial azimuthal anisotropy reflects the turbulent initial condition. At freeze-out, multiple narrow spikes formed from directed transverse shocks (volcanoes) are superimposed on top of the broader initial state anisotropy. The overall magnitude of the transverse energy is reduced more in the ideal case.

Fig. 16: The event-averaged transverse energy distribution (upper panels) is approximately rotation invariant within the finite statistics. However, a clear enhancement of the transverse energy correlations is observed (lower panels). The initial state correlations (dashed curves) are typically less than 1% and the final dynamical correlation due to transverse shocks (solid curves) are strongest in the ideal case (lower left panel).

Fig. 17: The dependence of the $E_\perp$ correlation function on the soft beam-jet background and the mini-jet p_0 scale is shown. The solid curves correspond to the default HIJING $p_0 = 2$ GeV/c and $E_\perp^{soft} \equiv \mathrm{d}E_\perp^{soft}/\mathrm{d}y = 0.5$ TeV shown in Fig. 16. For the ideal gas case (left panels) the dashed and dotted curves correspond to $p_0 = 2$ GeV/c and $E_\perp^{soft} = 0.25, 1.0$ TeV, respectively. For the first order transition case (right panels) the dashed curves correspond to $p_0 = 2$ GeV/c and $E_\perp^{soft} = 0.25$ TeV and the dotted curves to $p_0 = 1$ GeV/c and $E_\perp^{soft} = 1.0$ TeV, respectively. The dash–dotted curve corresponds to $p_0 = 1$ GeV/c and $E_\perp^{soft} = 0.5$ TeV. The lower panels show the ratio of the final to initial correlation functions in the restricted $\Delta\phi < \pi/2$ domain.

Fig. 18: The evolution of the central core ($r_\perp < 3$ fm) mean energy density $\langle \mathcal{E} \rangle$ is compared for 3+1–dimensional (thick solid and dashed curves) and one–dimensional Bjorken expansion (thin dashed and dash–dotted curves) and different equations of state (thick solid: first order transition, thick dashed and thin dash–dotted: ideal gas, thin dashed: $p = 0$).

Fig. 19: The evolution of the mean and rms fluctuations of the central core energy density $\langle \mathcal{E} \rangle$ is shown for the first order transition case. The dashed curve is the same as in Fig. 18.

Fig. 20: Photon transverse momentum spectra at $y = \eta = 0$. Thick lines are the average spectra calculated from the hydrodynamical evolution of 10 (turbulent) HIJING events, thin lines are for the evolution of a homogeneous Bjorken cylinder of radius $R = 4$ fm with the same initial energy content. Dashed lines are obtained assuming the equation of state with phase transition, full lines are for the ideal gas equation of state.